Topology and its Applications 23 (1986) 117-128
North-Holland

REMAINDERS OF *H*-CLOSED EXTENSIONS*

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Received 9 October 1984
Revised 15 April 1985

The remainders of compactifications of Tychonoff spaces have been studied extensively during the last five decades, but, the remainders of H-closed extensions of Hausdorff spaces have not been studied in a concerted manner so far. In this paper, we show that two Hausdorff spaces X and Y have the same remainders in their H-closed extensions if there exists a perfect, irreducible and θ-continuous mapping from X onto Y. Further, a Hausdorff space Z is shown to be the remainder of some H-closed extension of a given space X if and only if there exists a perfect (not necessarily continuous) mapping from $\sigma X\backslash X$ onto Z. We also show that if X is locally H-closed and $|\sigma X\backslash X|\geq\aleph_0$, then the set of remainders of H-closed extensions of X contains all separable H-closed spaces. We give examples to illustrate the differences between the sets of remainders of compactifications and remainders of H-closed extensions of a fixed Tychonoff space.

AMS(MOS) Subj. Class.: Primary 54D25; secondary 54D40

absolutes H-closed
compact   remainders

§1. Preliminaries

All spaces under consideration are Hausdorff. A space Y is an *extension* of a space X provided that X is a dense subspace of Y. Two extensions Y and Z of a space X are called *equivalent* if there is a homeomorphism f from Y onto Z such that $f|_X=\iota_X$, the identity map on X. We shall identify two equivalent extensions of a space X. With this convention, the class $\mathbb{E}(X)$ of all Hausdorff extensions of X is a set and $|\mathbb{E}(X)|\leq|\boldsymbol{P}(\boldsymbol{P}(\boldsymbol{P}(\boldsymbol{P}(X))))|$, where $|A|$ (respectively, $\boldsymbol{P}(X)$) denotes the cardinality (respectively, the power set) of A. If Y is an extension of X, then the subspace $Y\backslash X$ of Y is called the *remainder* (or, *outgrowth*) of X in Y. There are basically three important problems about remainders.

Problem A: given Hausdorff (respectively, Tychonoff) spaces X and Y, when do X and Y have the same remainders in their H-closed extensions (respectively, compactifications)?

* This is a part of the dissertation written at the University of Kansas under the supervision of Professor Jack Porter. The author is extremely grateful to Professor Porter for his invaluable help and suggestions.

Problem B: given Hausdorff (respectively, Tychonoff) spaces X and Y, when is Y a remainder of X in some H-closed extension (respectively, compactification) of X?

Problem C: given classes $\mathscr{A}$ and $\mathscr{B}$ of Hausdorff (respectively, Tychonoff) spaces, when is every member of $\mathscr{B}$ a remainder in some H-closed extension (respectively, compactification) of every member of $\mathscr{A}$?

The remainders of compactifications of Tychonoff spaces have been studied extensively during the last five decades. However, the remainders of H-closed extensions of Hausdorff spaces have not been studied in generality. In what follows, we shall study the Problems A, B, and C for the remainders of H-closed extensions of Hausdorff spaces. Our main theorems in this direction are Theorems 2.3, 2.5, 2.6, 3.3 and 3.7.

In this section we provide some well known definitions and results which will be used in the sequel. For a space X, $\tau(X)$ denotes the topology on X and $\mathrm{RO}(X)$ denotes the complete Boolean algebra of all regular open subsets of X. If X is a Hausdorff space, then $\mathrm{RO}(X)$ forms an open base for a coarser Hausdorff topology τ_s on X. The space (X, τ_s), denotes by X_s, is called the *semiregularization* of X (see [17]) and $X = X_s$ if and only if X is semiregular. Also $\mathrm{RO}(X) = \mathrm{RO}(X_s)$. An *open filter* (respectively, filterbase) on a space X will always mean a filter (respectively, filterbase) in the lattice $\tau(X)$ of all open subsets of X. An *open ultrafilter* on X is an open filter on X which is a maximal (with respect to set inclusion) element in the family of all open filters on X. If $\mathscr{F}$ is a filterbase on X, then $\mathrm{ad}_X(\mathscr{F}) = \bigcap\{\mathrm{cl}_X F\colon F \in \mathscr{F}\}$ denotes the *adherence* of $\mathscr{F}$ in X. A filterbase $\mathscr{F}$ is called *free* if $\mathrm{ad}_X(\mathscr{F}) = \emptyset$; otherwise, $\mathscr{F}$ is called *fixed.* If $\mathscr{A}$ is any nonempty family of open subsets of X having the finite intersection property then $\langle\mathscr{A}\rangle$ will denote the open filter on X generated by $\mathscr{A}$. For a space X, $\mathbb{F}(X)$ denotes the family of all free open ultrafilters on X.

Recall that a space X is called *H-closed* provided that X is closed in every Hausdorff space in which X is embedded. A space X is called *locally H-closed* if for each $x \in X$ there exists an open neighbourhood U of x such that $\mathrm{cl}_X U$ is H-closed. A subset A of a space X is called *relatively H-closed* if $\mathrm{cl}_X A$ is H-closed. A space X is called *minimal Hausdorff* if $\tau(X)$ does not contain any coarser Hausdorff topology of X. The *Katětov extension* [13] of a space X is the H-closed extension κX of X whose underlying set is the set $X \cup \mathbb{F}(X)$, with the topology $\tau(\kappa X)$ generated by the open base $\tau(X) \cup \{U \cup \{\mathscr{U}\}\colon U \in \mathscr{U} \in \mathbb{F}(X),\ U \in \tau(X)\}$. The *Fomin extension* [9] of a space X is the H-closed extension σX of X whose underlying set is the set $X \cup \mathbb{F}(X)$, with the topology $\tau(\sigma X)$ generated by the open base $\{o(U)\colon U \in \tau(X)\}$ where $o(U) = U \cup \{\mathscr{U} \in \mathbb{F}(X)\colon U \in \mathscr{U}\}$ (see [17] and [18] for more details).

In what follows for a Hausdorff (respectively, Tychonoff) space X, $\mathbb{H}(X)$ (respectively, $\mathbb{K}(X)$) will denote the set of all H-closed extensions (respectively, Hausdorff compactifications) of X. For a Tychonoff space Z, βZ denotes the Stone–Čech compactification of Z. Moreover, if X is Hausdorff and Z is Tychonoff, then

$\mathbb{R}_h(X)=\{hX\backslash X\colon hX\in\mathbb{H}(X)\}$, and $\mathbb{R}_c(Z)=\{cZ\backslash Z\colon cZ\in\mathbb{K}(Z)\}$. $\mathbb{N}$, $\mathbb{Q}$, and $\mathbb{R}$ will denote the spaces of natural numbers, the rationals and the reals, respectively.

1.1. Definitions. (a). *Two Hausdorff spaces X and Y are called $\mathbb{R}_h$-equivalent if* $\mathbb{R}_h(X)=\mathbb{R}_h(Y)$.

(b) *Two Tychonoff spaces X and Y are called $\mathbb{R}_c$-equivalent if* $\mathbb{R}_c(X)=\mathbb{R}_c(Y)$.

§2

In this section we discuss the conditions for two Hausdorff spaces X and Y to have the same remainders in their H-closed extensions, thereby answering Problem A. Recall that a mapping $f\colon X\to Y$ from a space X into a space Y is called *θ-continuous at a point* $x\in X$ [9] if for each open neighborhood G of $f(x)$ in Y there is an open neighborhood U of x in X such that $f(\mathrm{cl}_X U)\subseteq \mathrm{cl}_Y(G)$. If f is θ-continuous at each $x\in X$, then f is called *θ-continuous.* A map $f\colon X\to Y$ is called *compact* if for each $y\in Y$, the set $f^{\leftarrow}(y)=\{x\in X\colon f(x)=y\}$ is compact in X; f is called *perfect* if f is both a closed and a compact mapping. A map $f\colon X\to Y$ is called *irreducible* if f is onto and for each proper closed subset A of X, $f(A)\neq Y$. If $f\colon X\to Y$ is a mapping and $A\subseteq X$, then the *small image of A under f* is defined to be the set $f^{\#}(A)=\{y\in Y\colon f^{\leftarrow}(y)\subseteq A\}$ [16]. A surjection $f\colon X\to Y$ is closed and irreducible if and only if for each nonempty open subset U of X, $f^{\#}(U)$ is a nonempty open subset of Y. The facts listed in 2.1 below will be used subsequently; we refer the reader to [6] and [16] for details.

2.1. Facts. *Let $f\colon X\to Y$ be a closed, irreducible and θ-continuous surjection.*

(a) *If* $U\in\tau(X)$, *then* $\mathrm{int}_X\,\mathrm{cl}_X\,U=\mathrm{int}_X[f^{\leftarrow}(\mathrm{cl}_Y(f^{\#}(U)))]$ *and* $f(\mathrm{cl}_X\,U)=\mathrm{cl}_Y f^{\#}(U)$.

(b) *For each* V, W *in* $\mathrm{RO}(Y)$, $\mathrm{int}_X\,\mathrm{cl}_X f^{\leftarrow}(V\cap W)=\mathrm{int}_W\,\mathrm{cl}_X f^{\leftarrow}(V)\cap \mathrm{int}_X\,\mathrm{cl}_X f^{\leftarrow}(W)$.

(c) *For each* $W\in\tau(Y)$, $\mathrm{int}_X f^{\leftarrow}(\mathrm{cl}_Y W)=\mathrm{int}_X\,\mathrm{cl}_X f^{\leftarrow}(W)$.

(d) *If f is also perfect, then for each U in* $\mathrm{RO}(X)$, $f^{\#}(U)\in\mathrm{RO}(Y)$. *Moreover, in this case, if $\mathscr{F}$ is an open filter on X, then $f^{\#}(\mathscr{F})=\{f^{\#}(F)\colon F\in\mathscr{F}\}$ is an open filterbase on Y and* $\mathrm{ad}_Y\langle f^{\#}(\mathscr{F})\rangle=\mathrm{ad}_Y f^{\#}(\mathscr{F})=f(\mathrm{ad}_X(\mathscr{F}))$.

We will need the following result of Dow and Porter.

2.2. Proposition [7]. *Let hY be an H-closed extension of a space Y and suppose that Z is a space such that there is a continuous bijection $f\colon Z\to hY\backslash Y$. Then, there exists an H-closed extension T of Y such that $T\backslash Y$ is homeomorphic to Z.*

2.3. Theorem. *Let X and Y be Hausdorff spaces and f a perfect, irreducible and θ-continuous mapping from X onto Y. Then, for each $hX\in\mathbb{H}(X)$ there exists a space $hY\in\mathbb{H}(Y)$ such that $hX\backslash X$ is homeomorphic to $hY\backslash Y$.*

Proof. For each $p \in hX \backslash X$, let $F(p) = \langle f^{\#}(\mathcal{O}^p_{hX})\rangle$ be the open filter on Y generated by the filterbase $f^{\#}(\mathcal{O}^p_{hX})$, where $\mathcal{O}^p_{hX} = \{G \cap X\colon G \text{ open in } hX, p \in G\}$. Let $eY = Y \cup \{F(p)\colon p \in hX \backslash X\}$. For each $v \in \tau(Y)$, define $o_{eY}(V) = V \cup \{F(p)\colon V \in F(p)\}$. Then, the family $\{o_{eY}(V)\colon V \in \tau(Y)\}$ forms an open base for a Hausdorff topology on eY, and eY is a strict extension of the space Y (in the sense of Banaschewski [1]). Moreover, $\mathrm{ad}_Y(F(p)) = f(\mathrm{ad}_X(\mathcal{O}^p_{hX}))$ for each p. Define a map $g\colon hX \to eY$ by

$$g(p) = \begin{cases} F(p), & p \in hX \backslash \mathscr{X}, \\ f(p), & p \in X. \end{cases}$$

Then g is a surjection. We show that g is θ-continuous, proving thereby that eY is a H-closed extension of Y. To this end, we first show that if U is open in X, then

(i) $\quad g(o_{hX}(U) \backslash X) \subseteq o_{eY}(f^{\#}(U)) \backslash f^{\#}(U)$, and

(ii) $\quad g(\mathrm{cl}_{hX}(U) \backslash X) \subseteq \mathrm{cl}_{eY}(f^{\#}(U))$.

Now, (i) follows from the fact that if $p \in o_{hX}(U) \backslash X$, then $f^{\#}(U) \in f^{\#}(\mathcal{O}^p_{hX})$, whence, $g(p) \in o_{eY}(f^{\#}(U)) \backslash f(X)$. To prove (ii), let $p \in \mathrm{cl}_{hX}(o_{hX}(U)) \backslash X$, and let $o_{eY}(B)$, B open in Y, be a basic open neighborhood of $g(p) = F(p)$ in eY. Then $B \in F(p)$ and, so, there is an open subset A of X with $A \in \mathcal{O}^p_{hX}$ and $f^{\#}(A) \subseteq B$. Also, for every open neighborhood W of p in hX, $W \cap o_{hX}(U) \neq \emptyset$. Hence, $(W \cap X) \cap U \neq \emptyset$. In particular, $A \cap U \neq \emptyset$. Thus, $o_{eY}(B) \cap f^{\#}(U) \neq \emptyset$, whence, $g(p) \in \mathrm{cl}_{eY}(f^{\#}(U))$ and (ii) follows. To prove that g is θ-continuous, let $x \in X$ and let $o_{eY}(V)$, $V \in \tau(Y)$ be a basic open neighborhood of $g(x) = f(x)$ in eY. Since f is θ-continuous, there exists an open neighborhood W of x in X such that $f(\mathrm{cl}_X W) \subseteq \mathrm{cl}_Y V \subseteq \mathrm{cl}_{eY}(o_{eY}(V))$. Thus, $o_{hX}(W)$ is an open neighborhood of x in hX such that

$$\begin{aligned} g(\mathrm{cl}_{hX}(o_{hX}(W)) &= g(\mathrm{cl}_{hX}(W)) = g(\mathrm{cl}_{hX}(W) \backslash X) \cup g(\mathrm{cl}_X W) \\ &= g(\mathrm{cl}_{hX}(W) \backslash X) \cup f(\mathrm{cl}_X W) \subseteq g(\mathrm{cl}_{hX}(W) \backslash X) \\ &\quad \cup \mathrm{cl}_{eY}(o_{eY}(V)) \\ &\subseteq \mathrm{cl}_{eY}(f^{\#}(W)) \cup \mathrm{cl}_{eY}(o_{eY}(o_{eY}(V)) \end{aligned}$$

(by (ii)) $\subseteq \mathrm{cl}_{eY}(f(W)) \cup \mathrm{cl}_{eY}(o_{eY}(V)) = \mathrm{cl}_{eY}(o_{eY}(V))$. Thus, g is θ-continuous at x. If $p \in hX \backslash X$ and $o_{eY}(V)$, $V \in \tau(Y)$, is a basic open neighborhood of $g(p)$ in eY, then there is an open set $W \in \mathcal{O}^p_{hX}$ such that $f^{\#}(W) \subseteq V$. The same reasoning as above shows that $g(\mathrm{cl}_{hX}(o_{hX}(W))) \subseteq \mathrm{cl}_{eY}(o_{eY}(V))$, whence g is θ-continuous at p. Hence, g is θ-continuous, and eY is H-closed.

Now, let $g_1 = g|_{hX \backslash X}$. Then, g_1 is a bijection from $hX \backslash X$ onto $eY \backslash Y$. If $g(p) = F(p) \in eY \backslash Y$ and $o_{ey}(V)$ is a basic open neighborhood of $g(p)$, then $V \in F(p)$ and, so, there exists an open subset U of X such that $U \in \mathcal{O}^p_{hX}$ and $f^{\#}(U) \subseteq V$. Consequently, $o_{hX}(U) \backslash X$ is an open neighborhood of p in $hX \backslash X$ such that (by (i)) $g_1(o_{hX}(U) \backslash X) \subseteq o_{eY}(f^{\#}(U)) \backslash f(X) \subseteq o_{eY}(V) \backslash Y$. Thus, g_1 is a continuous bijection from $hX \backslash X$ onto $eY \backslash Y$. Therefore, by Proposition 2.2, there exists a H-closed extension hY of Y such that $hX \backslash X \cong hY \backslash Y$, and the theorem follows. □

2.4. Lemma. *Let hY be an H-closed extension of a space Y. For each $p \in hY\setminus Y$, let $\mathcal{O}^p_s = \{V \in \tau(Y)\colon V \supseteq \text{int}_Y \text{cl}_Y(W)$ for some $W \in \mathcal{O}^p_{hY}\}$, where $\mathcal{O}^p_{hY} = \{G \cap Y\colon G$ open in hY, $p \in G\}$. Let h^*Y be hY as a set, and, for each $U \in \tau(Y)$, let $o^*(U) = U \cup \{p \in hY\setminus Y;\ U \in \mathcal{O}^p_s\}$. Then:*

(a) *the family $\{o^*(U)\colon U \in \tau(Y)\}$ is an open base for a topology τ^* on h^*Y, and (h^*Y, τ^*) is an H-closed extension of Y. Moreover,*

(b) *the map $\iota'_{hY\setminus Y}\colon hY\setminus Y \to h^*Y\setminus Y$, where $\iota\colon hY \to h^*Y$ is the identity map, is a continuous bijection.*

Proof. The space h^*Y is easily seen to be a Hausdorff extension of Y. We show that h^*Y is H-closed. Let $\mathcal{F}$ be an open filter on h^*Y. Then, $\mathcal{F}_* = \{F \cap Y\colon F \in \mathcal{F}\}$ is an open filter on Y and, hence, $\text{ad}_{hY}(\mathcal{F}_*) \neq \emptyset$. If $p \in \text{ad}_{hY}(\mathcal{F}_*)\setminus Y$, let $o^*(U)$ be a basic open neighborhood of p in h^*Y. Then, U is open in Y and there is an open set $W \in \mathcal{O}^p_{hY}$ such that $\text{int}_Y \text{cl}_Y(W) \subseteq U$. Since $p \in \text{ad}_{hY}(\mathcal{F}_*)$, $W \cap G \neq \emptyset$ for each $G \in \mathcal{F}_*$. So, $o^*(U) \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. Therefore, $p \in \text{ad}_{h^*Y}(\mathcal{F})$. If $p \in \text{ad}_{hY}(\mathcal{F}_*) \cap Y$, let $o^*(W)$ be any open neighborhood of p in h^*Y. Then, $W \cap G \neq \emptyset$ for each $G \in \mathcal{F}_*$. So, $\text{int}_Y \text{cl}_Y(W) \cap G \neq \emptyset$ for all $G \in \mathcal{F}_*$. Hence, $o^*(W) \cap F \neq \emptyset$ for all $F \in \mathcal{F}$, and $p \in \text{ad}_{h^*Y}(\mathcal{F})$. Thus, by [23, 17K–L] h^*Y is H-closed, and (a) follows. To prove (b), let $o^*(U)\setminus U$, $U \in \tau(Y)$, be a basic open subset of $h^*Y\setminus Y$, and let $p \in o^*(U)\setminus U$. Then, there exists a regular open set $V \in \mathcal{O}^p_{hY}$ such that $V \subseteq U$. So, $p \in o^*(V)\setminus V \subseteq o^*(U)\setminus U$. Now, let $q \in o_{hY}(V)$. Then, $V \in \mathcal{O}^q_s$, $o_{hY}(V)\setminus V \in \tau(hY\setminus Y)$ and $p \in o_{hY}(V)\setminus V \subseteq o^*(V)\setminus V \subseteq o^*(U)\setminus U$. This proves (b). $\square$

2.5. Theorem. *Let X and Y be Hausdorff spaces and let f be a perfect, irreducible and θ-continuous mapping from X onto Y. Then, for each H-closed extension hY of Y, there is an H-closed extension hX of X such that $hX\setminus X$ is homeomorphic to $hY\setminus Y$.*

Proof. Let h^*Y be the H-closed extension of Y constructed in Lemma 2.4. For each $p \in h^*Y\setminus Y$, the family $\mathcal{B}^p = \{\text{int}_X \text{cl}_X(f^{\leftarrow}(U))\colon U \in \mathcal{O}^p_s$, U regular open in $Y\}$ is an open filter base on X, and if $\mathcal{F}^p = \langle\mathcal{B}^p\rangle$ is the open filter on X generated by $\mathcal{B}^p$, then $\text{ad}_Y(\mathcal{O}^p_s) = f(\text{ad}_X(\mathcal{F}^p))$. Let $hX = X \cup \{p\colon p \in h^*Y\setminus Y\}$, and for each open subset U of X, let

$$\hat{o}(U) = U \cup \{p \in hX\setminus X\colon U \in \mathcal{F}^p\}.$$

Then, the family $\{\hat{o}(U)\colon U \in \tau(X)\}$ is an open base for a Hausdorff topology $\hat{\tau}$ on hX, and hX is an extension of X. We show that hX is H-closed. Let $\mathcal{U}$ be an open filter on hX, and let $\mathcal{U}_* = \{U \cap X\colon U \in \mathcal{U}\}$. Then, since h^*Y is H-closed, the open filterbase $f^{\#}(\mathcal{U}_*)$ on Y has a nonempty adherence in h^*Y. Let $p \in \text{ad}_{h^*Y}(f^{\#}(\mathcal{U}_*))$. If $p \in h^*Y\setminus Y$, then for each regular open set $W \in \mathcal{O}^p_s$ of Y, $W \cap f^{\#}(V) \neq \emptyset$ for each $V \in \mathcal{U}_*$. Therefore, $\text{int}_X \text{cl}_X(f^{\leftarrow}(W)) \cap f^{\leftarrow}(f^{\#}(V)) \neq \emptyset$, whence, $\text{int}_X \text{cl}_X(f^{\leftarrow}(W)) \cap V \neq \emptyset$ for all $V \in \mathcal{U}_*$. Consequently, if $\hat{o}(G)$ is any basic open neighborhood of p in h^*X, then $G = \hat{o}(G) \cap X \in \mathcal{F}^p$ and $G \supseteq \text{int}_X \text{cl}_X(f^{\leftarrow}(W))$ for some $W \in \mathcal{O}^p_s \cap \text{RO}(Y)$. Thus, $\hat{o}(G) \cap V \neq \emptyset$ for all $V \in \mathcal{U}_*$, whence, $p \in \text{ad}_{hX}(\mathcal{U})$. If $p \in Y$, let G

be an open subset of X containing $f^{\leftarrow}(p)$. Then, $f^{\#}(G)$ is an open neighborhood of p in Y. Therefore, $f^{\#}(G)\cap f^{\#}(V)\neq\emptyset$ for all $V\in\mathcal{U}_*$, and hence $G\cap V\neq\emptyset$ for all $V\in\mathcal{U}_*$. This again leads to the conclusion that $p\in \mathrm{ad}_{hX}(\mathcal{U})$. Thus, $\mathrm{ad}_{hX}(\mathcal{U})\neq\emptyset$ and, by [23, 17K–L], hX is H-closed extension of X.

Now, let $g: h^*Y\backslash Y\to hX\backslash X$ be the map given by $g(p)=p$, $p\in h^*Y\backslash Y$. Let $\hat{o}(U)\backslash U$, $U\in\tau(X)$, be a basic open neighborhood of $p=g(p)$ in hX. Then, $U\in\mathcal{F}^p$ and, so, there is a regular open subset $W\in\mathcal{O}_s^p$ of Y such $U\supseteq \mathrm{int}_X\, \mathrm{cl}_X f^{\leftarrow}(W)$. For each $q\in o^*(W)\backslash W$ (where $o^*(W)$ is defined in Lemma 2.4), $W\in\mathcal{O}_s^p$, $\mathrm{int}_X\,\mathrm{cl}_X(f^{\leftarrow}(W))\in\mathcal{F}^q$ and $U\in\mathcal{F}^q$. Therefore, $g(q)=q\in\hat{o}(U)\backslash U$. Thus, g is a continuous bijection from $h^*Y\backslash Y$ onto $hX\backslash X$. Consequently, by Lemma 2.4(b), $g\circ \iota|_{hY\backslash Y}$ is a continuous bijection from $hY\backslash Y$ onto $hX\backslash X$, and the theorem follows from Proposition 2.2. □

Combining the results of Theorems 2.3 and 2.5, we get the following theorem:

2.6. Theorem. *Let X and Y be Hausdorff spaces, and let $f: X\to Y$ be a perfect, irreducible and θ-continuous mapping from X onto Y. Then, X and Y are $\mathbb{R}_h$-equivalent.*

Recall that the *Iliadis absolute* [12] of a Hausdorff space X is the unique (upto homeomorphism) space EX such that EX is extremely disconnected and zero-dimensional and there exists a perfect, irreducible, θ-continuous mapping k_X from EX onto X. The underlying set of EX is the set of all the convergent open ultrafilters $\mathcal{U}$ on X and the family $\{O_XU: U\in\tau(X)\}$, where $O_XU=\{\mathcal{U}\in EX: U\in\mathcal{U}\}$, is an open base for the topology $\tau(EX)$. Moreover, the map $k_X: EX\to X$ is given by $k_X(\mathcal{U})=\mathrm{ad}_X(\mathcal{U})$. Two spaces are called *coabsolute* if they have homeomorphic absolutes. The *Hausdorff absolute* [2, 15] of a space X is the space PX whose underlying set is the set of EX and whose topology $\tau(PX)$ is generated by the open base $\{O_XU\cap k_X^{\leftarrow}(V): U, V\in\tau(X)\}$. For more information about EX and PX we refer the reader to [16, 17, 19]. We remark that $EX=(PX)_s$ and the space PX is unique (upto homeomorphism) with respect to possessing these properties: PX is extremely disconnected (but not necessarily zero-dimensional) and there exists a perfect, irreducible and continuous mapping $k_X: PX\to X$ given by $k_X(\mathcal{U})=\mathrm{ad}_X(\mathcal{U})$, $\mathcal{U}\in PX$. We now state some immediate corollaries of Theorem 2.6.

2.7. Corollary. *Let X be a Hausdorff space and K a H-closed space. Then, $\mathbb{R}_h(X)=\mathbb{R}_h(EX)=\mathbb{R}_h(PX)=\mathbb{R}_h(X\oplus K)$, where $X\oplus K$ denotes the topological direct sum of X and K.*

2.8. Corollary. *Any two locally compact, noncompact, metric spaces X and Y without isolated points and having the same density character are $\mathbb{R}_h$-equivalent.*

Proof. The spaces X and Y are coabsolute [10, p. 255]. Therefore, the result follows from Corollary 2.7. □

2.9. Corollary. *Let X and Y be locally compact paracompact spaces such that there is a one to one mapping from $K(X)$ onto $K(Y)$ which preserves '$\vee$' and '$\wedge$', where $K(Z)$ denotes the collection of regular closed compact subsets of a space Z. Then, X and Y are $\mathbb{R}_h$-equivalent.*

Proof. By [10, p. 255], the spaces X and Y are coabsolute. Therefore the result follows from Corollary 2.7. $\square$

We close this section with the remark that the result of Corollary 2.8 is a stronger version in the $\mathbb{R}_h$-setting than the result of Cain–Chandler–Faulkner [3, Theorem 5.4] in the $\mathbb{R}_c$-setting.

§3

In this section, for two Hausdorff spaces, we give a necessary and sufficient condition for one of the spaces to be the remainder of an H-closed extension of the other space. Also, we show that there is a large class of spaces which are remainders of H-closed extensions. We first recall that if hX is an H-closed extension of a Hausdorff space X, then there is a continuous mapping κ_h from κX onto hX that leaves X pointwise fixed [17, 1.7]. We shall denote by σ_h the composition map $\kappa_h \circ \iota$, where $\iota : \sigma X \to \kappa X$ is the identity map, and refer to it as the map induced by κ_h. Also, there exists a perfect, irreducible and θ-continuous map π_h from βEX onto hX [19, 2.3]. It is proved in [18, 5.3] that $\pi_\sigma^* = \pi_\sigma|_{\beta EX \setminus EX}$ is a homeomorphism from $\beta EX \setminus EX$ onto $\sigma X \setminus X$. These facts will be used in the sequel.

3.1. Lemma. *Let hX be a H-closed extension of a Hausdorff space X. Let $\sigma_h : \sigma X \to hX$ be the map induced by κ_h, and let $\sigma_h^* = \sigma_h|_{\sigma X \setminus X}$. Then, $\sigma_h^* : \sigma X \setminus X \to hX$ is a perfect surjection.*

Proof.

$$\begin{tikzcd}
\beta EX \arrow[r, "\pi_\sigma"] \arrow[dr, "\pi_h"'] & \sigma X \arrow[r, "\iota"] \arrow[d, "\sigma_h"] & \kappa X \arrow[dl, "\kappa_h"] \\
& hX &
\end{tikzcd}$$

It suffices to show that there is a perfect map from $\beta EX \setminus EX$ onto $hX \setminus X$. Let $\pi_\beta : \beta EX \to \sigma X$ and $\pi_h : \beta EX \to hX$ be the perfect, irreducible and θ-continuous surjections [19, 3.4]. Then, $\sigma_h \circ \pi_\sigma = \pi_h$. Let A be a closed subset of $\beta EX \setminus EX$. Then, $B = \mathrm{cl}_{\beta EX}(A)$ is a closed subset of βEX, and $B \setminus EX = A$. Now, $\pi_h(B) = \pi_h[\mathrm{cl}_{\beta EX}(A) \setminus EX] \cup \pi_h[\mathrm{cl}_{\beta EX}(A) \cap EX] = \pi_h(A) \cup \pi_h[\mathrm{cl}_{\beta EX}(A) \cap EX]$. Since $\pi_h(B)$ is closed in hX and $\pi_h(B) \setminus X$ is closed in $hX \setminus X$, it follows that $\pi_h^* = \pi_h|_{\beta EX \setminus EX} : \beta EX \setminus EX \to hX \setminus X$ is a closed mapping. Obviously, π_h^* is a compact surjection. Let $f=(\pi_\sigma^*)^{\leftarrow}$. Then $f: \sigma X\backslash X \to \beta EX\backslash EX$ is a homeomorphism and $\sigma_h^* = \pi_h^* \circ f$. Thus, σ_h^* is a perfect mapping from $\sigma X\backslash X$ onto $hX\backslash X$, and the result follows. □

3.2. Corollary. *Let X be a Tychonoff space. Then, X is Čech-complete if and only if $\sigma X\backslash X$ is a F_σ-set in X.*

We now prove the main theorem of this section, thereby answering Problem B.

3.3. Theorem. *Let X and Y be Hausdorff spaces. Then Y is homeomorphic to $hX\backslash X$ for some H-closed extension hX of X if and only if there is a perfect (not necessarily continuous) mapping from $\sigma X/X$ onto Y.*

Proof. If $Y \cong hX\backslash X$ for some H-closed extension hX of X, then, by Lemma 3.1, there exists a perfect mapping from $\sigma X\backslash X$ onto Y.

To prove the converse, let $f: \sigma X\backslash X \to Y$ be a perfect surjection. Then, for each $p\in Y$, $f^{\leftarrow}(p)$ is a compact subset of $\sigma X\backslash X$, and hence a compact subset of σX. Therefore, $\boldsymbol{P}=\{f^{\leftarrow}(p): p\in Y\}\cup\{\{x\}: s\in X\}$ is a partition of σX by compact sets. Hence, by [17], there exists an H-closed extension eX of X which induces the same partition $\boldsymbol{P}$ on σX. In fact, eX is the quotient space $\kappa X/R$, where R is the equivalence relation on κX induced by the partition $\boldsymbol{P}$. Let $\pi_e: \beta EX \to eX$ be the perfect, irreducible and θ-continuous surjection. Then, by a theorem of Vermeer and Wattel [21, 2.3], the family $\{\pi_e(A): A$ closed in $\beta EX\}$ is a closed basis for a minimal Hausdorff topology on eX. Denote eX with this (new coarser) minimal Hausdorff topology by Z. Then, $\pi_e: \beta EX \to Z$ is a perfect, irreducible and θ-continuous surjection. By [13] (see also [17, 1.6]), $(eX)_s = Z$ and, hence, Z is an H-closed extension of X_s. Therefore, by Theorem 2.3, there is a H-closed extension bX of X such that $bX\backslash X = Z\backslash X_s$. We show that there is a continuous bijection from Y onto $Z\backslash X_s$. Let σ_Z^*: $\sigma X\backslash X \to Z\backslash X_s$ be the perfect map given by Lemma 3.1. Define $g: Z\backslash X_s \to Y$ by $g\circ\sigma_Z^* = f$. Then, obviously, g is one to one and onto. Let B be a basic closed subset of $Z\backslash X_s$. Then, $B=\pi_e(A)\cap(Z\backslash X_s) = \pi_e(A\backslash EX)$, where A is some closed subset of βEX. Now, $\pi_\sigma(A\backslash EX)$ is a closed subset of $\sigma X\backslash X$, and, by Lemma 3.1, $\pi_e(A\backslash EX) = \sigma_Z^*\circ\pi_\sigma(A\backslash EX)$. So, $g(B) = g\circ\pi_e(A\backslash EX) = g\circ\sigma_Z^*\circ\pi_\sigma(A\backslash EX) = f\circ\pi_\sigma(A\backslash EX)$. Since f is perfect, $f\circ\pi_\sigma(A\backslash EX)$ is a closed subset of Y, and, hence, $g(B)$ is closed in Y. Since g is one to one, g is a closed mapping. Hence, $g^{\leftarrow}: Y\to Z\backslash X_s(=bX\backslash X)$ is a continuous bijection. Therefore, by Proposition 2.2, there is a H-closed extension hX of X such that $Y=hX\backslash X$. □

3.4. Corollary. *Let X and Y be Hausdorff spaces. Then, $\mathbb{R}_h(Y)\subseteq\mathbb{R}_h(X)$ if and only if there exists a perfect mapping from $\sigma X\backslash X$ onto $\sigma Y\backslash Y$.*

Proof. If $\mathbb{R}_h(Y)\subseteq\mathbb{R}_h(X)$, then there exists an H-closed extension hX of X such that $hX\backslash X = \sigma Y\backslash Y$. By Lemma 3.1, there exists a perfect mapping from $\sigma X\backslash X$

onto $hX\backslash X$, and hence onto $\sigma Y\backslash Y$. Conversely, let $f:\sigma X\backslash X\to\sigma Y\backslash Y$ be a perfect surjection. Let $hY\backslash Y$ be in $\mathbb{R}_h(Y)$ for some $hY\in\mathbb{H}(Y)$, and let $\sigma_h^*:\sigma Y\backslash Y\to hY\backslash Y$ be the perfect surjection given by Lemma 3.1. Then $\sigma_h^*\circ f$ is a perfect mapping from $\sigma X\backslash X$ onto $hY\backslash Y$. Hence, by Theorem 3.3, $hY\backslash Y=hX\backslash X$ for some $hX\in\mathbb{H}(X)$, and the result follows.

3.5. Corollary. *Two spaces X and Y are $\mathbb{R}_h$-equivalent if and only if there exist a perfect mapping from $\sigma X\backslash X$ onto $\sigma Y\backslash Y$ and a perfect mapping from $\sigma Y\backslash Y$ onto $\sigma X\backslash X$. Moreover, if X and Y are $\mathbb{R}_h$-equivalent, then $|\sigma X\backslash X|=|\sigma Y\backslash Y|$.*

3.6. Proposition. *Let X be a locally H-closed space such that $|\sigma X\backslash X|\geq\aleph_0$. Then, $\beta\mathbb{N}\in\mathbb{R}_c(EX)$. In particular, $\beta\mathbb{N}\in\mathbb{R}_h(EX)=\mathbb{R}_h(X)$, and $|\sigma X\backslash X|\geq 2^c$, where c is the cardinality of the continuum.*

Proof. The space EX is locally compact, extremally disconnected, and βEX contains the infinite closed set $\beta EX\backslash EX$. By [8, VII.2.2], there exists a family $\{U_n: n\geq 1\}$ of clopen subsets of βEX such that $U_n\cap U_m=\emptyset$ for $n\neq m$, and $U_n\cap(\beta EX\backslash EX)\neq\emptyset$ for each $n=1,2,3,\ldots$. Let $U_0=\beta EX\backslash \mathrm{cl}_{\beta EX}(\bigcup_{n\geq 1}U_n)$. Then, U_0 is a clopen subset of EX disjoint from each U_n. Let $D=U_0\cup\bigcup_{n\geq 1}U_n$. Then, D is an open and dense subset of βEX such that $\beta D=\beta EX$. Define a mapping $f:D\to\mathbb{N}$ by $f(U_n)=\{n\}$ for each $n=1,2,3,\ldots$, and $f(U_0)=\{1\}$. Then, f is a continuous surjection. Let $f^\beta:\beta EX\to\beta\mathbb{N}$ be the Stone-Čech extension of f. Since, $f^\beta(\beta EX\backslash EX)\supseteq\mathbb{N}$ and $\beta EX\backslash EX$ is compact, it follows that $\beta\mathbb{N}=\mathrm{cl}_{\beta\mathbb{N}}(\mathbb{N})\subseteq f^\beta(\beta EX\backslash EX)\supseteq\beta\mathbb{N}$, whence, $f^\mu(\beta EX\backslash EX)=\beta\mathbb{N}$. Hence, by a theorem of Magill [14, 2.1], $\beta\mathbb{N}\in\mathbb{R}_c(EX)\subseteq\mathbb{R}_h(EX)$. The result now follows by Corollary 2.7. □

3.7. Corollary. *Let X be a locally H-closed Hausdorff space such that $|\sigma X\backslash X|\geq\aleph_0$. If Z is any separable H-closed space, then $Z\in\mathbb{R}_h(X)$.*

Proof. The space EZ is a separable and compact Hausdorff space. Let A be a countable dense subset of EZ, and let $g:\mathbb{N}\to A$ be any bijection. Since $\mathbb{N}$ is discrete, g is continuous. Let $g^\beta:\beta\mathbb{N}\to EZ$ be the Stone-Čech extension of g. Now, by Proposition 3.6, $\beta\mathbb{N}\in\mathbb{R}_h(X)$. Hence, by Theorem 3.3, there is a perfect mapping from $\sigma X\backslash X$ onto $\beta\mathbb{N}$. If k_Z is the usual map from EZ onto Z, then $k_Z\circ g^\beta\circ\phi$ is a perfect mapping from $\sigma X\backslash X$ onto Z. Hence, by Theorem 3.3, $Z\in\mathbb{R}_h(X)$.

3.8. Note. Let X be a locally H-closed space such that $|\sigma X\backslash X|\geq\aleph_0$. Let Z be any one of the following spaces:

(a) $Z=\sigma\mathbb{N}$, $\sigma\mathbb{Q}$, or $\sigma\mathbb{R}$,
(b) Z is a perfect image of a separable H-closed space,
(c) Z is a countable H-closed space, or
(d) Z is any compact metric space.

Then, $Z\in\mathbb{R}_h(X)$.

3.9. Theorem. *Let E be a locally compact, extremally disconnected space such that E contains an infinite closed discrete subset. Then, $\beta\mathbb{N} \in \mathbb{R}_c(E)$. In particular, every separable compact Hausdorff space is a $\mathbb{R}_c(E)$.*

Proof. Let A be an infinite, closed, discrete subset of E. By extracting a countably infinite subset from A, we may assume that A contains a closed copy N of $\mathbb{N}$. By [11, Ex.6M, 9H], N is C^*-embedded in E. Let $f_0: N \to [0, 1]$ be any map. Since f_0 is continuous, there exists a continuous extension $f: E \to [0, 1]$ of f_0. Let f^β be the Stone-Čech extension of f mapping βE continuously into $[0, 1]$, and let $g = f^\beta|_{\mathrm{cl}_{\beta E}(N)}$. Then, g is a continuous extension of f_0 mapping $\mathrm{cl}_{\beta E}(N)$ into $[0, 1]$. It then follows that N is C^*—embedded in $\mathrm{cl}_{\beta E}(N)$. Hence, by [22, Prop. 1.48], $\beta N = \mathrm{cl}_{\beta E}(N)$. Further, since N is closed in E, $(E \setminus N) \cap \mathrm{cl}_{\beta E}(N) = \emptyset$. So, $\beta N \setminus N = \mathrm{cl}_{\beta E}(N) \setminus N \subseteq \beta E \setminus E$. In particular, it follows that $|\beta E \setminus E| = |\sigma E \setminus E| > \aleph_0$. The result now follows from Proposition 3.6.

3.10. Remark. (a) Let X be a locally H-closed, noncountably H-closed space. Then, EX is locally compact, non-countably compact space. By [8, XI.3.2], EX contains an infinite, closed, discrete subset. By Theorem 3.9, $|\sigma X \setminus X| = |\beta EX \setminus EX| \geq 2^c$. Thus, if Z is any one of the spaces given in Note 3.8, then $Z \in \mathbb{R}_h(X)$.

(b) Let X be a Hausdorff space. If there is a compact Hausdorff space Z such that $Z \in \mathbb{R}_h(X)$, then X is locally H-closed. It is unknown whether the word 'compact' can be replaced with the word 'H-closed'.

3.11. Proposition. *Suppose X is a locally H-closed Hausdorff space such that $|\sigma X \setminus X| \geq \aleph_0$. Then, there is an H-closed extension hX of X such that $|hX \setminus X| = \aleph_0$.*

Proof. Since βEX is a regular space containing the infinite subset $\beta EX \setminus EX$, by [8, VII.2.4], there is a sequence $\{U_n: n \geq 0\}$ of open subsets of βEX such that $\mathrm{cl}_{\beta EX}(U_m) \cap \mathrm{cl}_{\beta EX}(U_n) = \emptyset$ for each $n \neq m$, and $(\beta EX \setminus EX) \cap \mathrm{cl}_{\beta EX}(U_n) \neq \emptyset$ for each $n \geq 1$. Let $C_n = (\beta EX \setminus EX) \cap \mathrm{cl}_{\beta EX}(U_n)$, and let $C = (\beta EX \setminus EX) \setminus \bigcup_{n \geq 1} C_n$. Then, the family $\boldsymbol{P} = \{C_n: n \geq 1\} \cup \{C\}$ is a compact partition of $\beta EX \setminus EX = \sigma X \setminus X$. Therefore, by [17], there is a H-closed extension hX of X such that $|hX \setminus X| = |\boldsymbol{P}|$, and the theorem follows. □

We close this section with a few remarks, and we omit their straightforward proofs. For definitions, we refer the reader to Walker [22].

3.12. Remarks. (a) Let X be a Hausdorff space which admits an infinite, locally finite collection of κ nonempty relatively H-closed open subsets. Then, the cellularity of $\sigma X \setminus X$ is at least $\kappa^{\aleph_0}$.

(b) If X is a locally H-closed, noncountably H-closed space, then the cellularity of $\sigma X \setminus X$ is at least c.

(c) If X is not countably H-closed, then $\sigma X \setminus X$ is not homogeneous.

§4

In this section we shall give examples illustrating the differences between the remainders of compactifications and the remainders of H-closed extensions of Tychonoff spaces, and show that these remainders differ in a wide variety of ways.

4.1. Example. Let ω_1 be the first uncountable ordinal, and let $I(\omega_1)$ denote the set of all the isolated points of ω_1. Then, $I(\omega_1)$ is dense in ω_1, and $|I(\omega_1)| = |\omega_1| = \aleph_1$. Let $\mathcal{U}$ be a uniform ultrafilter on $I(\omega_1)$. Let $\mathcal{V} = \{V \subseteq \omega_1\colon V \text{ is open in } \omega_1, I(\omega_1) \cap V \in \mathcal{U}\}$. Then, $\mathcal{V}$ is a free open ultrafilter on ω_1 (associated with $\mathcal{U}$). If $\mathcal{U}'$ is any other uniform ultrafilter on $I(\omega_1)$ distinct from U and $\mathcal{V}'$ is the free open ultrafilter on ω_1 associated with $\mathcal{U}'$, then $\mathcal{V} \neq \mathcal{V}'$. Conversely, suppose that $\mathcal{U}$ is a free open ultrafilter on ω_1. Let $\mathcal{U}_* = \{U \cap I(\omega_1)\colon U \in \mathcal{U}\}$. Then, $\mathcal{U}_*$ is an ultrafilter on $I(\omega_1)$. We show that $\mathcal{U}_*$ is a uniform ultrafilter. Assume that there exists a set $\tilde{U} \in \mathcal{U}_*$, where $\tilde{U} = U \cap I(\omega_1)$, $U \in \mathcal{U}$, such that $|\tilde{U}| \leq \aleph_0$. Then, $\tilde{U} \subseteq [0, a]$ for some $\alpha < \omega_1$, and $\mathrm{cl}_{\omega_1}(U) = \mathrm{cl}_{\omega_1}(\tilde{U}) \subseteq \mathrm{cl}_{\omega_1}([0, \alpha]) = [0, \alpha]$, which is a compact subset of ω_1. From this it follows that $\mathcal{U}$ is fixed, leading to a contradiction. Hence, $\mathcal{U}_*$ is a uniform ultrafilter on $I(\omega_1)$. Since distinct free open ultrafilters on ω_1 have distinct traces on $I(\omega_1)$, we have shown that there is a one to one correspondence between the collection of all the free open ultrafilters on ω_1 and the collection $\underline{u}(I(\omega_1))$ of all uniform ultrafilters on $I(\omega_1)$. Thus, by [5; Cor. 7.8], $|\beta EX \backslash EX| = |\sigma X \backslash X| = 2^{2^{\aleph_1}}$. However, $|\beta\omega_1 \backslash \omega_1| = 1$. Thus, there does not exist any *continuous mapping from* $\beta\omega_1 \backslash \omega_1$ onto $\beta E\omega_1 / E\omega_1$, whence, $\mathbb{R}_c(\omega_1) \neq \mathbb{R}_c(E\omega_1)$. We note, however, that $\mathbb{R}_c(\omega_1) \subset \mathbb{R}_c(E\omega_1)$, and (by Corollary 2.7) $\mathbb{R}_h(\omega_1) = \mathbb{R}_h(E\omega_1)$.

The following is an example of two spaces X and Y such that $\mathbb{R}_c(X) = \mathbb{R}_c(Y)$, but $\mathbb{R}_h(X) \neq \mathbb{R}_h(Y)$.

4.2. Example. Let ω_1 denote the first uncountable ordinal of cardinality $\aleph_1$, and let ω_r be the initial ordinal of cardinality $(2^{\aleph_1})^+$, where α^+ denotes the successor cardinal of α. Then $|\beta\omega_1 \backslash \omega_1| = 1 = |\beta\omega_r \backslash \omega_r|$, and hence $\mathbb{R}_c(\omega_1) = \mathbb{R}_c(\omega_r)$. However, as shown in Example 4.1, $|\sigma\omega_1 \backslash \omega_1| = 2^{2^{\aleph_1}}$, and $|\sigma\omega_r \backslash \omega_r| = 2^{2^{\aleph_r}}$. By Cantor's theorem, since $2^{\aleph_1} < \aleph_r$, it follows that $2^{2^{\aleph_1}} < 2^{2^{\aleph_r}}$. It follows that there does not exist any (perfect) map from $\sigma\omega_1 \backslash \omega_1$ onto $\sigma\omega_r \backslash \omega_r$, and hence, by Corollary 3.5, $\mathbb{R}_h(\omega_1) \neq \mathbb{R}_h(\omega_r)$.

The following is an example of two locally connected, locally compact separable, metrizable spaces X and Y such that $\mathbb{R}_h(X) = \mathbb{R}_h(Y)$, but $\mathbb{R}_c(X) \neq \mathbb{R}_c(Y)$.

4.3. Example. Let $X = [0, \infty)$ regarded as a subspace of the reals with usual topology. A routine verification shows that $\beta X \backslash X$ is connected. Now, let $Y = X_1 \oplus X_2$, where X_1 and X_2 are disjoint copies of X. Then, X and Y are both locally connected, locally compact, metrizable spaces with the same density character. Neither X, nor

Y has isolated points. Thus, by Corollary 2.8, $\mathbb{R}_h(X)=\mathbb{R}_h(Y)$. Now, the natural projection $g: X_1 \oplus X_2 \to X$ is a perfect and continuous mapping from Y onto X. Hence, $\mathbb{R}_c(X) \subset \mathbb{R}_c(Y)$. However, since $\beta X \backslash X$ is connected and $\beta Y \backslash Y$ is disconnected, there can be no continuous mapping from $\beta X \backslash X$ onto $\beta Y \backslash Y$. Therefore, $\mathbb{R}_c(X) \neq \mathbb{R}_c(Y)$.

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